Short Papers

Dispersion in Unilateral Finlines on Anisotropic Substrates

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Abstract —**The dispersion characteristics of the dominant and higher order modes in unilateral finlines on uniaxially anisotropic substrates have been obtained. The solution has been obtained by applying the equivalent transmission-line concept in the spectral domain and by using Galerkin's method. Numerical results for the propagation constant as a function of the slot-width ratio and frequency are presented.**

I. Introduction

Interest in finline circuits and components is growing rapidly in view of increasing demands of microwave integrated and millimeter-wave circuits. Different types of finline configurations, such as unilateral, bilateral, and antipodal on isotropic substrates, have been extensively studied and employed in practice [1]–[3]. While finlines on crystalline substrates, ferrites, and semiconductors have received considerable attention in recent years [4]–[6], little work appears to have been done on finlines on anisotropic substrates. For isotropic dielectric substrates such as fused silica and alumina, the assumption of isotropy is only an approximation. Substrate anisotropy could have important implications on the operation of microstrip circuits [6], [7], especially at higher millimeter-wave frequencies. Therefore, it is desirable to study in detail the effect of the dielectric anisotropy on the dispersion characteristics of finlines.

This paper presents an analysis of wave propagation along unilateral finlines on anisotropic substrates. The method of solution is based on the spectral-domain immittance approach suggested by Itoh [8].

II. Formulation of the Problem

Consider a unilateral finline on a uniaxially anisotropic dielectric substrate (medium 1) inside a hollow metal rectangular waveguide of $2a \times 2b$ cross section. Let the Cartesian coordinate system (Fig. 1(a)) be so chosen that its z-axis, along which the wave is assumed to be propagating with a propagation constant β, is parallel to the axis of the waveguide and its y-axis, which is normal to the plane of the finline (the E plane), coincides with the optical axis of the anisotropic dielectric substrate. An $\exp(j\omega t)$ time dependence of the field quantities is assumed in the following analysis and is omitted throughout the paper. Further, let the anisotropy in region 1 be described by the following permittivity tensor:

$$\epsilon = \epsilon_0 \begin{bmatrix} \epsilon_t & 0 & 0 \\ 0 & \epsilon_y & 0 \\ 0 & 0 & \epsilon_t \end{bmatrix} \tag{1}$$

where ϵ_t and ϵ_y are the relative dielectric constants in the directions perpendicular and parallel to the optical axis.

Since the finline being considered supports a hybrid field, both the TM-to-y and the TE-to-y modes would be present. Let the Fourier transform (over the variable x, denoted by quantities with tilde over them) of the y-field components be defined as

$$\tilde{E}_y(\alpha, y) = \int_{-b}^{b} E_y(x, y)\, e^{j\alpha x}\, dx \tag{2a}$$

$$\tilde{H}_y(\alpha, y) = \int_{-b}^{b} H_y(x, y)\, e^{j\alpha x}\, dx \tag{2b}$$

where α takes discrete values given by $\alpha = n\pi/b$ for odd modes, including the dominant one (electric wall at $x = 0$), and by $\alpha = \{n + 1/2\}\pi/b$ for even modes (magnetic wall at $x = 0$) with $n = 0, \pm 1, \pm 2, \pm 3, \cdots$. $\tilde{E}_y(\alpha, y)$ and $\tilde{H}_y(\alpha, y)$ now satisfy Helmholtz equations in the Fourier domain:

$$\left[\frac{d^2}{dy^2} - \frac{\epsilon_t}{\epsilon_y}\left(\alpha^2 + \beta^2 - k_0^2\epsilon_y\right)\right]\tilde{E}_y(\alpha, y) = 0 \tag{3a}$$

$$\left[\frac{d^2}{dy^2} - \left(\alpha^2 + \beta^2 - k_0^2\epsilon_t\right)\right]\tilde{H}_y(\alpha, y) = 0 \tag{3b}$$

where $k_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the free-space propagation constant.

The other field components can be expressed in terms of y-field components. The corresponding inverse transform of (2a) is

$$E_y(x, y)\, e^{-j\beta z} = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{E}_y(\alpha, y)\, e^{-j(\alpha x + \beta z)}\, d\alpha \tag{4}$$

which may be put in the form

$$E_y(x, y)\, e^{-j\beta z} = \frac{1}{2b}\sum_{n=-\infty}^{\infty} \tilde{E}_y(\alpha, y)\, e^{-j\beta' v} \tag{5}$$

where $\beta' = \sqrt{\alpha^2 + \beta^2}$. Similar expressions may be obtained for $H_y(x, y)$. Introducing the Cartesian coordinate system (u, v, y), defined by

$$u = z\sin\theta - x\cos\theta \tag{6a}$$

$$v = z\cos\theta + x\sin\theta \tag{6b}$$

where $\theta = \tan^{-1}(\alpha/\beta)$, it may be noted from (5) that the fields are expressed as a superposition of inhomogeneous plane waves propagating along the v direction with phase constant β'. Thus, the plane waves in the v direction are decomposed into TM-to-y $(\tilde{E}_y, \tilde{E}_v, \tilde{H}_u)$ and TE-to-y $(\tilde{H}_y, \tilde{E}_u, \tilde{H}_v)$ fields. The effect of the fin metallization in the $y = t + d$ plane is taken into account by introducing current densities $\tilde{J}_v$ and $\tilde{J}_u$ [2] that generate the TM and the TE field, respectively. Therefore, equivalent transmission lines can be introduced so that the current densities and electric

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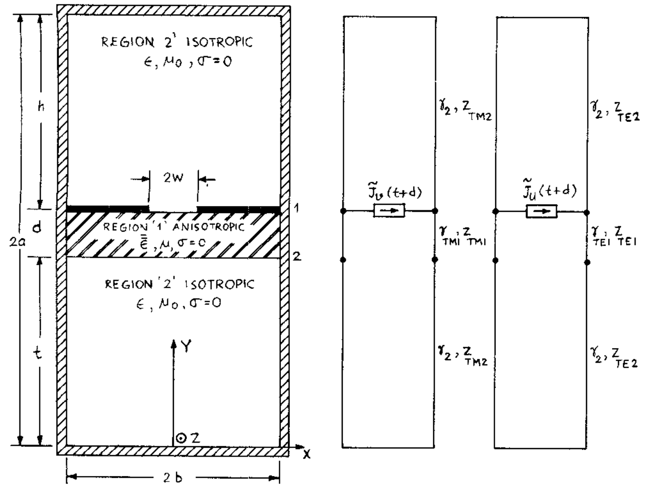

(a) (b)

Fig. 1. (a) Cross-sectional view of a unilateral finline on an anisotropic substrate. (b) Equivalent transmission lines for the TM-to-y and TE-to-y waves.

fields are related by

$$\tilde{J}_v(\alpha, t+d) = Y_{11}^e \tilde{E}_v(\alpha, t+d) \tag{7}$$

$$\tilde{J}_u(\alpha, t+d) = Y_{11}^h \tilde{E}_u(\alpha, t+d). \tag{8}$$

Using (7) and (8), the strip current densities and the tangential electric slot field components are related to each other in the (x, z) coordinate system by

$$\begin{bmatrix} Y_{zz}^{11} & Y_{zx}^{11} \\ Y_{xz}^{11} & Y_{xx}^{11} \end{bmatrix} \begin{bmatrix} \tilde{E}_z(\alpha, t+d) \\ \tilde{E}_x(\alpha, t+d) \end{bmatrix} = \begin{bmatrix} \tilde{J}_z(\alpha, t+d) \\ \tilde{J}_x(\alpha, t+d) \end{bmatrix} \tag{9}$$

where

$$Y_{zz}^{11} = Y_{11}^e \cos^2\theta + Y_{11}^h \sin^2\theta \tag{10}$$

$$Y_{zx}^{11} = Y_{xz}^{11} = \left(Y_{11}^e - Y_{11}^h\right)\sin\theta\cos\theta \tag{11}$$

$$Y_{xx}^{11} = Y_{11}^e \sin^2\theta + Y_{11}^h \cos^2\theta. \tag{12}$$

Here, Y_{11}^e and Y_{11}^h are the driving-point admittances at the interface 1 (see Fig. 1(a)) for the TM and TE modes, respectively, and are given by

$$Y_{11}^e = Y_{2u}^e + Y_{\mathrm{TM1}} \frac{Y_{\mathrm{TM1}} + Y_{2L}^e \coth\gamma_{\mathrm{TM1}} d}{Y_{2L}^e + Y_{\mathrm{TM1}} \coth\gamma_{\mathrm{TM1}} d} \tag{13a}$$

$$Y_{2L}^e = Y_{\mathrm{TM2}} \coth\gamma_2 t \tag{13b}$$

$$Y_{2u}^e = Y_{\mathrm{TM2}} \coth\gamma_2 h \tag{13c}$$

$$Y_{11}^h = Y_{2u}^h + Y_{\mathrm{TE1}} \frac{Y_{\mathrm{TE1}} + Y_{2L}^h \coth\gamma_{\mathrm{TE1}} d}{Y_{2L}^h + Y_{\mathrm{TE1}} \coth\gamma_{\mathrm{TE1}} d} \tag{14a}$$

$$Y_{2L}^h = Y_{\mathrm{TE2}} \coth\gamma_2 t \tag{14b}$$

$$Y_{2u}^h = Y_{\mathrm{TE2}} \coth\gamma_2 h. \tag{14c}$$

Y_{TM1} and Y_{TE1} are, respectively, the TM and TE admittances in the y direction in region 1 (the anisotropic region) and are given by

$$Y_{\mathrm{TM1}} = \frac{\tilde{H}_{u1}}{\tilde{E}_{v1}} = \frac{j\omega\epsilon_0\epsilon_t}{\gamma_{\mathrm{TM1}}} \tag{15}$$

$$Y_{\mathrm{TE1}} = \frac{\tilde{H}_{v1}}{\tilde{E}_{u1}} = \frac{\gamma_{\mathrm{TE1}}}{j\omega\mu_0} \tag{16}$$

where γ_{TM1} and γ_{TE1} are the propagation constants in the y direction for the TM and TE waves, respectively, in the region 1 and are obtained from (3) as

$$\gamma_{\mathrm{TM1}} = \sqrt{\frac{\epsilon_t}{\epsilon_y}\left(\alpha^2 + \beta^2 - k_0^2\epsilon_y\right)} \tag{17}$$

$$\gamma_{\mathrm{TE1}} = \sqrt{\alpha^2 + \beta^2 - k_0^2\epsilon_t}. \tag{18}$$

Expressions for Y_{TM2}, Y_{TE2}, and γ_2 in region 2 (the isotropic region) are given by

$$Y_{\mathrm{TM2}} = \frac{j\omega\epsilon_0\epsilon_r}{\gamma_2} \tag{19}$$

$$Y_{\mathrm{TE2}} = \frac{\gamma_2}{j\omega\mu_0} \tag{20}$$

$$\gamma_2 = \sqrt{\alpha^2 + \beta^2 - k_0^2\epsilon_r}. \tag{21}$$

Returning to (9), the unknown aperture fields $\tilde{E}_z$ and $\tilde{E}_x$ are expanded in terms of known basis functions, and Galerkin's method in the spectral domain, along with Parseval's theorem, is applied. This results in a homogeneous eigenvalue matrix equation [8]. The dispersion relation for the dominant and higher order modes is then derived by solving for the eigenvalues of β that render the matrix determinant zero at a particular frequency f, slot-width ratio w/b, substrate thickness d, and permittivity tensor $\bar{\bar{\epsilon}}$.

III. Numerical Results

The series expansions of the slot fields in the spectral domain are given by

$$\tilde{E}_z(\alpha) = \sum_m a_m \tilde{f}_{zm}(\alpha) \tag{22}$$

$$\tilde{E}_x(\alpha) = \sum_m b_m \tilde{f}_{xm}(\alpha) \tag{23}$$

where $\tilde{f}_{zm}(\alpha)$ and $\tilde{f}_{xm}(\alpha)$ are Fourier transforms of the basis functions $f_{zm}(x)$ and $f_{xm}(x)$, which are chosen to be zero except for $|x| < w$.

The choice of the basis functions is important to achieve a highly efficient numerical solution, high computational speed, and convergence. Therefore, sinusoidal functions, as modified by an "edge condition" term [2], that satisfy the boundary conditions and whose Fourier transforms are available in closed forms, are used as the basis functions in this paper. They are

$$\{f_{zm}(x), f_{xm}(x)\} = \frac{\{\sin,\cos\}\{m\pi(x-w)/2w\}}{\sqrt{1-\left[\dfrac{x}{w}\right]^2}} \qquad |x| < w$$

$$= 0 \qquad \text{otherwise} \tag{24}$$

where $m = 0, 2, 4, \cdots$ for odd modes and $1, 3, 5, \cdots$ for even modes. (25)

The effective dielectric constant $\epsilon_{\mathrm{eff}} = \beta^2/k_0^2$ for the dominant and higher order modes of the finline structures has been numerically computed for the slot in the central position. In all calculations, region 2 has been considered to be free space so that its $\epsilon_r = 1$. Numerical computation has been carried out for finline structures consisting of WR-28 waveguide (26.5–40-GHz band) and a substrate of thickness 0.125 mm located in the central E plane of the waveguide. The substrates considered are Epsilam-10

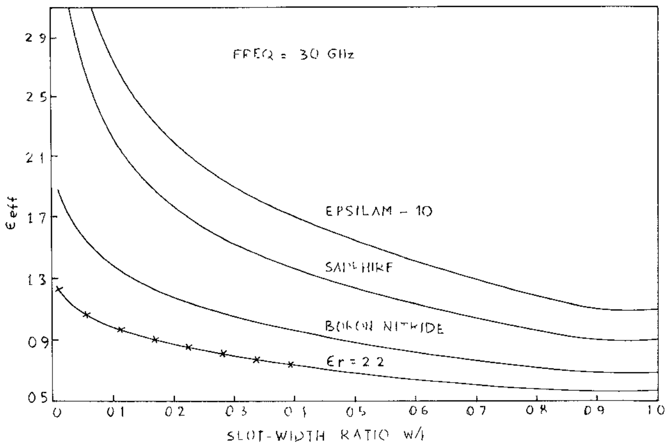

Fig. 2. Slot-width dependence of the effective dielectric constant for the dominant mode for a finline on isotropic and anisotropic substrates in the WR-28 waveguide. $2a = 7.112$ mm; $2b = 3.556$ mm; $d = 0.125$ mm; $t = h = 3.4935$ mm. —— calculated results. × × × × × × Schmidt *et al.* [2].

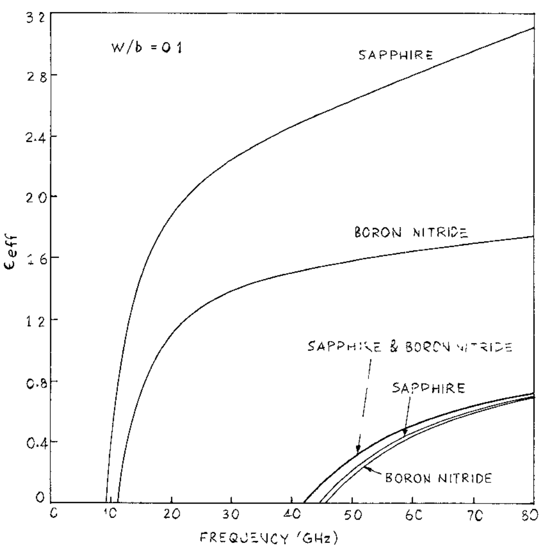

Fig. 3. Dispersion characteristics of the first three odd modes for a finline on sapphire and boron nitride substrates. $2a = 7.112$ mm; $2b = 3.556$ mm; $d = 0.125$ mm; $t = h = 3.4935$ mm.

($\epsilon_t = 13, \epsilon_y = 10.2$), sapphire ($\epsilon_t = 9.4, \epsilon_y = 11.6$), and boron nitride ($\epsilon_t = 5.12, \epsilon_y = 3.4$).

The results of the numerical computation are presented in Figs. 2 and 3. Fig. 2 shows the effective dielectric constant ϵ_{eff} for the dominant mode as a function of a complete range of slot-width ratios for a unilateral finline in the WR-28 waveguide on isotropic as well as anisotropic substrates at a frequency of 30 GHz. Results for the isotropic dielectric case, $\epsilon_r = 2.2$, are given for the sake of comparison with those obtained by Schmidt *et al.* [2], and very good agreement is observed.

Fig. 3 shows the frequency dependence of the effective dielectric constant of the first three odd modes for a finline in the WR-28 waveguide on sapphire and boron nitride substrates with a slot-width ratio of 0.1.

IV. Conclusions

The unilateral finline on an electrically anisotropic substrate has been analyzed using the equivalent transmission-line concept in the spectral domain in conjunction with Galerkin's procedure. The substrate used is a uniaxial anisotropic substrate. Numerical results have been obtained for the dominant and higher order modes for finlines in the WR-28 waveguide. Numerical results for finlines on isotropic dielectric substrate have also been obtained and compared with the results available in the literature.

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Waveform Standards for Electrooptics: A Pulse Duration Comparison

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***Abstract*—A transfer standard has been developed for use in comparing the measurement capability of the Automatic Pulse Measurement System (APMS) at the National Bureau of Standards to that of the recently developed electrooptic samplers. This transfer standard is a comb generator driven by a 90-MHz sine wave. Using this standard, measurements were made of the pulse waveform of a comb generator output with both the APMS and an electrooptic sampler. A comparison was then made of the pulse duration (full width at half maximum) obtained in the two waveform measurements. The result was a duration of 102 ps as measured by the APMS and 112 ps as measured by the electrooptic sampler. The signal-to-noise ratio at the comb generator input was improved over that of previous measurements, and a correction for pulse broadening was made to achieve this result. The pulse broadening was caused by the impedance mismatch between the sampler and the transmission system (50 Ω).**

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